COGNITIVE DATA SCIENCE
IN SUSTAINABLE COMPUTING

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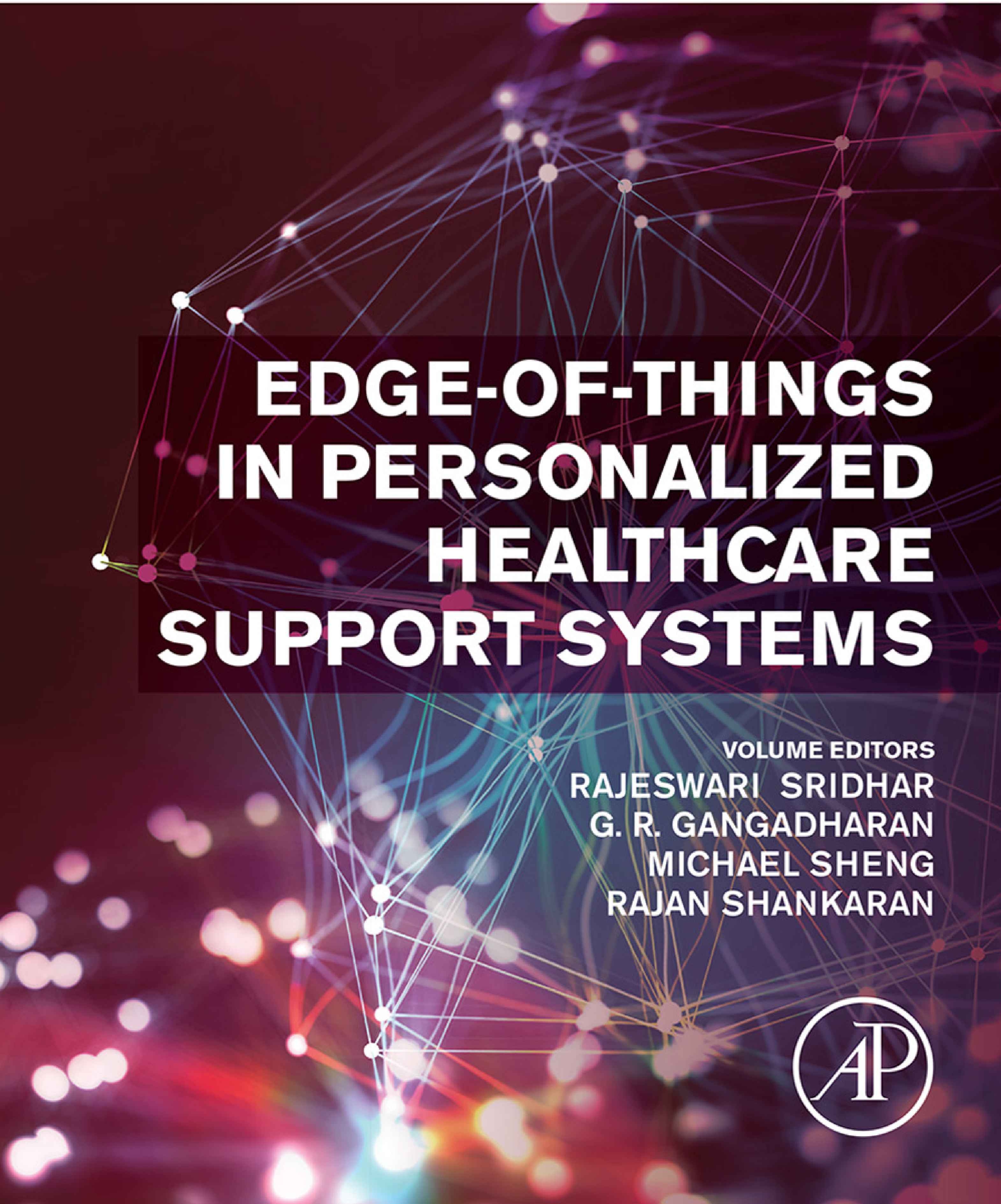

EDGE-OF-THINGS IN PERSONALIZED HEALTHCARE SUPPORT SYSTEMS

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CHAPTER 11

Feedback context-aware pervasive systems in healthcare management: a Boolean Network approach

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11.1 Introduction and related works

The growing complexity of modern software systems stimulated the use of component-based approaches and the enforcement of the separation of concerns (Djoudi et al., 2016). In context-aware computing the separation is made between the functions the system is built for, that can change in time, owing to different conditions, and the context in which the system must operate, which sets the current environmental situation (Cardozo & Dusparic, 2020; Li et al., 2020).

Context-aware databases have been used in different application domains, for example, to tailor application relevant data sets, to reduce information noise and increase the precision of information retrieval algorithms; to build smarter application environments; and to benefit from newly discovered web services. To better adapt to the specific problem, different models of the context have been proposed (see, e.g., Bolchini et al., 2007a, 2007b; Bolchini et al., 2009; Schreiber et al., 2012).

Among the most widely used definitions of context and of context-aware computing, those proposed in Dey (2001) state: "Context is any information that can be used to characterize the situation of an entity. An entity is a person, place, or object that is considered relevant to the interaction between a user and an application, including the user and applications themselves." and "A system is context-aware if it uses context to provide relevant information and/or services to the user, where relevancy depends on the user's task."

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New application domains such as self-adapting systems (Cardozo & Dusparic, 2020; Schreiber & Panigati, 2017), safety-critical applications, autonomous vehicle design and manufacturing (Tran et al., 2012), disaster prevention, or healthcare management, require a very high level of data quality and dependability that can only be achieved by formally determining their behaviors. Properties such as:

- the existence of *stable equilibrium points*;
- the absence of *undesired* oscillations (limit cycles);
- *observability*—the measure of how well internal states of a system can be inferred from the knowledge of its external outputs (and, possibly, of the corresponding data inputs);
- *controllability*—the capability of an external input data set (the vector of control variables) to drive the internal state of a system from any initial state to any other final state in a finite number of steps; and
- *reconstructability*—when the knowledge of the input and output vectors in a discrete time interval allows to uniquely determine the system final state

are only some of the features, together with the existence of *fault detection and identification* algorithms, which allow to guarantee the expected and safe operation of a system.

Several approaches to the formal definition/validation/verification of pervasive, context-aware, and self-adapting systems have been proposed; bigraphs and model-checking approaches using linear temporal logic have been proposed in Cardozo and Dusparic (2020), Cherfia et al. (2014), Djoudi et al. (2016), Li et al. (2020), Serral et al. (2010), Shehzad et al. (2004), Sindico and Grassi (2009), Tran et al. (2012), and Wang et al. (2011).

In Arcaini et al. (2015), formal properties such as validation, verification, and system correctness of self-adapting systems for systems specified by MAPE-K control loops are discussed using abstract state machines. In Padovitz et al. (2004) and Padovitz et al. (2005) a state-space approach is adopted to model the *situation* dimension and to determine the likelihood of transitions between *situation subspaces*, while keeping the other context dimensions constant. The probability of a transition is evaluated by resorting to concepts analogous to those of velocity and acceleration, typically adopted for mechanical systems. Table 11.1 summarizes the relevant features of some of the cited approaches.

Systems theorists are well acquainted with the techniques to prove the aforementioned properties, and in Diao et al. (2005) the authors have

Table 11.1 Comparison of approaches.

Reference	Main issues	Target architecture	Models and tools	Application domains
Arcaini et al. (2015)	Modeling, validation, verification, correctness	Distributed C-A self-adaptive systems	MAPE-K, simulation, ASMETA model checking (LTL)	Smart home
Cardozo and Dusparic (2020)	Development	Adaptive systems	COP	City transport management
Cherfia et al. (2014)	Modeling and verification	C-A systems	Bigraphs	Smart home
Diao et al. (2005)	Control theory	Self-managing systems	DTAC	IBM server control
Djoudi et al. (2016)	Specification and verification	C-A adaptive systems	Model based, CTXs-Maude	Cruise control
Filieri et al. (2011)	Control theory, Reliability	Self-adapting software	Discrete time Markov chains (DTMC)	SW reliability
Li et al. (2020)	Validation	Cyber-physical systems	Model checking	Motion control
Nzekwa et al. (2010)	Stability	C-A self-adaptive	Algorithm composition	Temperature control
Padovitz et al. (2004)	Stability	C-A pervasive	State space	Smart home
Serral et al. (2010)	Development	C-A pervasive	Model driven, OWL, UML, PervML	Smart home
Shehzad et al. (2004)	Need of formal model	C-A systems	CAMUS, Ontologies	Smart home
Sindico and Grassi (2009)	Development	C-A systems	CAMEL, UML	Personnel employment
Tran et al. (2012)	Modeling and Verification	C-A adaptive systems	ROAD4	Cruise control
Wang et al. (2011)	Formalization of structure and behavior	C-A systems	Bigraphs	Academic work

explored "… the extent to which control theory can provide an architectural and analytic foundation for building self-managing systems…."

Since context-aware systems are digital and mostly based on logics (Filieri et al., 2011; Nzekwa et al., 2010), Boolean Control Networks (BCN) seem to be the appropriate framework in which context-aware systems can be formalized. In recent times, through the introduction of the semitensor product of matrices, the representative equations of a logic system have been converted into an equivalent algebraic form (Cheng & Qi, 2010a; 2010b), and solutions to problems such as controllability, observability, stability, and reconstructability have been proposed (Cheng & Qi, 2009; Fornasini & Valcher, 2013a; Fornasini & Valcher, 2016; Zhang & Zhang, 2016).

In a previous paper (Schreiber & Valcher, 2019), we have proposed the use of BCNs to model open-loop context-aware systems, and we have illustrated our approach by focusing on the interesting example of an early-warning hydrogeological system, in which inputs are gathered as data provided by a set of physical sensors and data provided as messages by public web services. For this model, to which lots of open-loop context-aware systems can be reduced, we proved (1) the existence of equilibrium points corresponding to constant inputs; (2) the absence of limit cycles; (3) its reconstructability from the output measurements; and (4) the detectability of stuck-in-faults. In this paper, *we extend this technique to closed-loop feedback systems*.

The used approach is, by all means, general, but we found it beneficial to illustrate it by referring to a healthcare management example. Healthcare systems, which involve the interaction among several components, can benefit from formal design and verification methods to enhance their safety and efficacy properties. To make it understandable and focus on the methodology, we have oversimplified the example that therefore must be regarded as a *proof of concept*, rather than a realistic model.

The paper is organized as follows. In Section 11.2, we describe the fundamental steps of a BCN-based verification methodology for context-aware feedback systems and introduce the basics of BCN formalism. In Section 11.3, we describe the data structures of the case study; then we model the context as well as the functional system such as BCNs, as explained in Section 11.4. In Section 11.5 the mathematical formalization of real-life requirements is presented, while Section 11.6 brings some conclusive remarks.

11.2 A Boolean Control Network-based methodology

The fundamental structure of a feedback control system is shown in Fig. 11.1. The general idea is the following one: given a reference value (or set point), to make a selected internal variable achieve the desired value, one can feed back the controller with the output measurements taken on the system. The difference between the set point and the output measurement will be used by the controller to generate the appropriate control action to be applied to the target system.

The first step in our methodology is the identification of the functional blocks that can be mapped on the scheme of Fig. 11.1 and to use a formal model, such as sequential machines, to represent them.

In the second step the obtained model is translated into the BCN formalism, and in the third step the system properties are formally checked.

11.2.1 Boolean Control Networks

Before proceeding, we introduce a few elementary notions regarding the left semitensor product and the algebraic representations of Boolean Networks (BNs) and BCN. The interested reader is referred to Cheng et al. (2011) for a general introduction to this class of models and their basic properties. Additional references for the specific properties and results we will use in the paper will be introduced in the following.

We consider Boolean vectors and matrices, taking values in $\mathcal{B} = \{0, 1\}$, with the usual logical operations (And $\wedge$, Or $\vee$, and Negation $\bar{\cdot}$). δ_k^i denotes the ith canonical vector of size k, namely, the ith column of the k-dimensional identity matrix I_k. $\mathcal{L}_k$ is the set of all k-dimensional canonical vectors, and $\mathcal{L}_{k \times n} \subset \mathcal{B}_{k \times n}$ the set of all $k \times n$ logical matrices, namely, $k \times n$ matrices whose n columns are canonical vectors of size k.

The (left) semitensor product $\ltimes$ between matrices (in particular, vectors) is defined as follows (Cheng et al., 2011): given $L_1 \in \mathcal{L}_{r_1 \times c_1}$

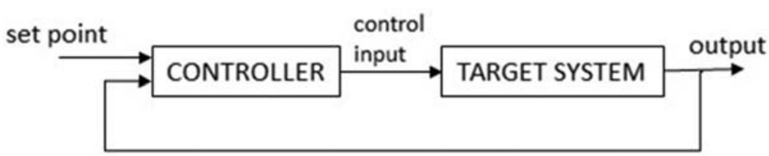


Figure 11.1 A feedback control system.

and $L_2 \in \mathcal{L}_{r_2 \times c_2}$, we set

$$L_1 \ltimes L_2 := (L_1 \otimes \mathrm{I}_{\mathrm{T}/c_1})(\mathrm{L}_2 \otimes \mathrm{I}_{\mathrm{T}/r_2}), \text{ with } T := \mathrm{l.c.m.}\{\mathrm{c}_1, \mathrm{r}_2\}.$$

The semitensor product generalizes the standard matrix product, meaning that when $c_1 = r_2$, then $L_1 \ltimes L_2 = L_1 L_2$. In particular, when $x_1 \in \mathcal{L}_{r_1}$ and $x_2 \in \mathcal{L}_{r_2}$, we have $x_1 \ltimes x_2 \in \mathcal{L}_{r_1 r_2}$.

A *BCN* is a logic state-space model taking the form:

$$\begin{aligned} X(t+1) &= f(X(t), U(t)), \\ Y(t) &= h(X(t), U(t)), t \in \mathbb{Z}_+, \end{aligned} \tag{11.1}$$

where $X(t), U(t)$, and $Y(t)$ are the n-dimensional state variable, the m-dimensional input variable, and the p-dimensional output variable at time t, taking values in $\mathcal{B}^{\mathrm{n}}$, $\mathcal{B}^{\mathrm{m}}$, and $\mathcal{B}^{\mathrm{p}}$, respectively. f and h are logic functions, that is, $f: \mathcal{B}^{\mathrm{n}} \times \mathcal{B}^{\mathrm{m}} \to \mathcal{B}^{\mathrm{n}}$, while $h: \mathcal{B}^n \times \mathcal{B}^m \to \mathcal{B}^p$. By making use of the semitensor product $\ltimes$, the BCN (11.1) can be equivalently represented as (Cheng et al., 2011):

$$\begin{aligned} x(t+1) &= L \ltimes u(t) \ltimes x(t), \\ y(t) &= H \ltimes u(t) \ltimes x(t), \quad t \in \mathbb{Z}_+, \end{aligned} \tag{11.2}$$

where $L \in \mathcal{L}_{N \times NM}$ and $H \in \mathcal{L}_{P \times NM}$, $N := 2^n, M := 2^m$ and $P := 2^p$. This is known as the *algebraic expression* of the BCN. The matrix L can be partitioned into M square blocks of size N, namely as

$$L = \begin{bmatrix} L_1 & L_2 & \cdots & L_M \end{bmatrix}.$$

For every $i \in \{1, 2, \cdots, M\}$ the matrix $L_i \in \mathcal{L}_{N \times N}$ represents the logic matrix that relates $x(t+1)$ to $x(t)$, when $u(t) = \delta_M^i$, namely

$$u(t) = \delta_M^i \Rightarrow x(t+1) = L_i x(t).$$

In the special case when the logic system has no input, its algebraic expression becomes

$$\begin{aligned} x(t+1) &= Lx(t), \\ y(t) &= Hx(t), \end{aligned} \tag{11.3}$$

and it is called *BN*.

It is easy to realize that the previous algebraic expressions (11.2) and (11.3) can be adopted to represent any state-space model in which the state, input and output variables take values in finite sets, and hence the sizes of the state, input and output vectors N, M, and P need not be powers of 2. When so, oftentimes BCNs and BNs are called *multivalued (control) networks* (Cheng et al., 2011). With a slight abuse of terminology, in this work we will refer to them as BCNs and BNs. Also, in the following, capital letters will be used to denote the original vectors/variables, taking values in finite sets, and the same lowercase letters will be used to denote the corresponding canonical vectors.

To focus on the ideas and on the modeling techniques, rather than on the Boolean math, in Section 11.3, we have chosen to address the model structure and properties without assigning specific numerical values to the logic matrices involved in the system description. Thus we have derived general results that can be tailored to the specific needs and choices of the application. We believe that this is the power of the proposed modeling approach: its flexibility and generality.

Finally, we provide a deterministic model of the patient health evolution, which represents the evolution of the average case of a patient affected by a specific form of illness. Accordingly, interpretations to the patient symptoms, as captured by the values of his/her vital parameters, are given and, based on them, well-settled medical protocols, to prescribe therapies and locations where such therapies need to be administered, are applied. A probabilistic model of the patient reaction to therapies, that also keeps into account the probabilistic correlation between actual health status and the measured values of his/her vital parameters, requires the use of Probabilistic BNs and will be the subject of our future research. Thus our case study must be considered as a proof of concept and not necessarily as representative of a real system.

11.3 The case study

We consider a multiple feedback loops system as it naturally arises when modeling the evolution of a patient's health status, subjected to medical therapies, whose vital parameters are, in turn, used as inputs to update the therapies to be administered to the patient.

The model provides the mathematical formalization of a possible data tailoring algorithm, running on the mobile device of a nurse in a hospital, *aimed at providing him/her with all and only the information on the therapies the patients in his/her ward are to be given*. Fig. 11.2 shows the overall tailoring process.

A hospital keeps a database that stores all the data relevant both to the patients and to the administrative, medical, and assistance employees.

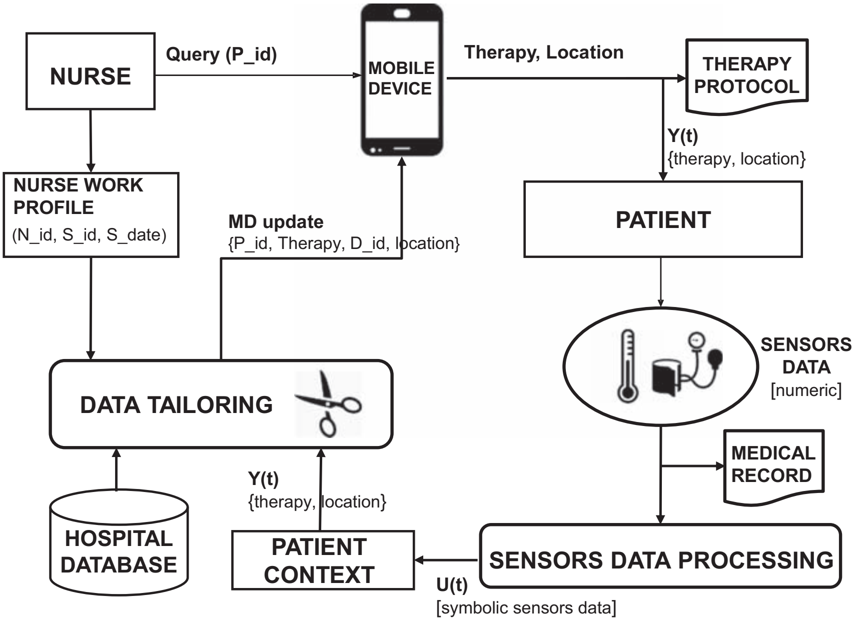


Figure 11.2 The tailoring process.

The work of a nurse is guided by an application on his/her mobile device. The app assists the nurse in his/her routine work following established medical protocols.

Each patient is provided with healthcare wearable sensors, measuring the variables that characterize his/her medical status, in our example: the body temperature (*bt*), the blood pressure (*bp*), and the heartbeat frequency (*hf*) (Baskar et al., 2020; Castillejo et al., 2013). For ease of representation, all of these variables are discretized and take values in the finite set $S = \{low, medium, high\}$. Therefore there are $3^3 = 27$ possible combinations (triples) of the sensors' symbolic data.

As detailed in Section 11.4, the patient context is constituted by a set of variables and it determines the therapy to be given (e.g., the drugs, their amount, and timing). The treatment should change the actual patient status—thus changing the sensors' output—and, possibly, could require a relocation of the patient in a different location, thus determining feedback loops. Moreover, the model is meant to be general and we intentionally neglect diagnose-prescription issues. We assume that *the therapies are effective and they will eventually lead the patient to be dismissed*. Fig. 11.3 shows the feedback schema of the case study.

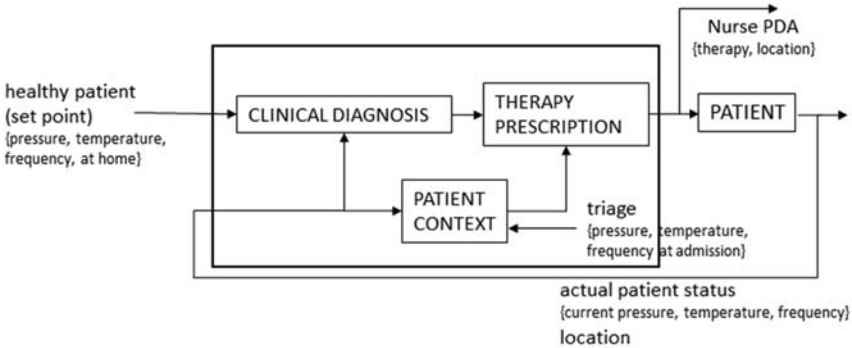
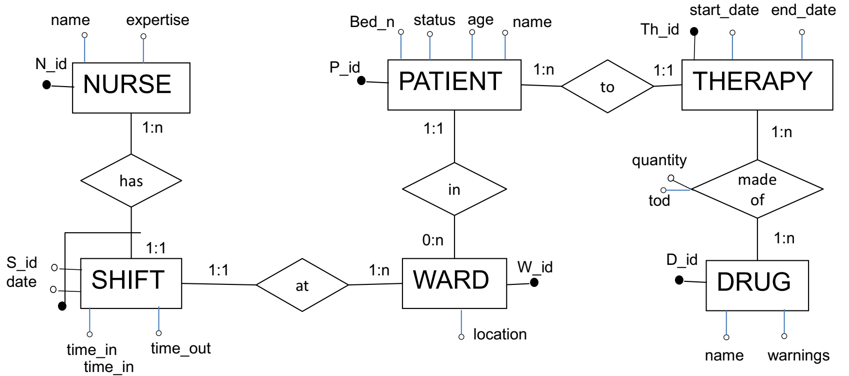


Figure 11.3 The case study general model.



ENTITIES

PATIENT (P_id, name, age, status, Bed_id, W_id)
NURSE (N_id, name, expertise)
SHIFT (N_id, S_id, date, time_in, time_out)
WARD (W_id, location)
DRUG (D_id, name, warnings)
THERAPY (Th_id, start_date, end_date, P_id)

RELATIONSHIPS

made_of (D_id, Th_id, time_of_day, quantity)

Figure 11.4 The hospital database.

Fig. 11.4 shows a portion of the schema of the hospital database, which must be dynamically tailored to store, on the mobile device of each nurse in a shift, all and only the treatments each patient in his/her ward is to be given in that shift. Treatments are defined in the therapy protocol adopted for the diagnosed illness. The numerical values coming from the sensors—registered in the medical record—are converted into their symbolic aggregate counterparts {*low; medium; high*} in the sensors data processing block and affect the estimated patient status, which can take five values: healthy (*H*), convalescent (*C*), under observation (*UO*), Ill (*I*), and life critical (*LC*). The estimated patient status determines the physician's decision on both the therapy and the patient location—at home (*h*), in the hospital ward (*hw*), in an intensive care unit (*ICU*). Of course, the prescribed therapies are also related to the current location and to the location recommended for the patient. For instance, some therapies can be given in a hospital *ICU* or in a *ward,* but cannot be given at *home.* On the other hand, the medical context can require a relocation of the patient. Thus data tailoring is made based on two different criteria:

- The work profile of the nurse, which is used to select all and only the patients he/she must attend; it is downloaded at the beginning of the shift and is not affected by external events (Listing 11.1).
- The medical status of the patient, which dynamically varies and hence requires different treatments.

Listing 11.1 Tailoring the nurse work profile.

```
select P_id, bed_n
   from nurse, shift, ward, patient
   where N_id = "A" AND S_id = "X" AND S_date = "yy/mm/dd"
```

The query on the nurse's mobile device is shown in Listing 11.2. In the rest of the paper, we focus only on medical and not on administrative issues. The schema of the tailored data, stored on the mobile device, is shown in Fig. 11.5.

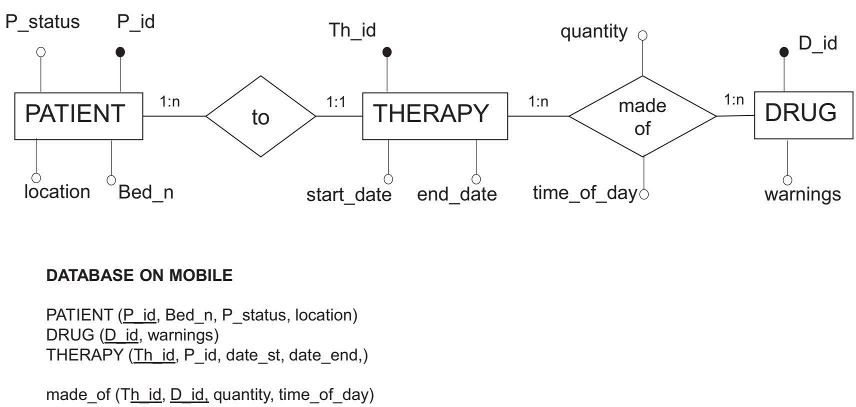


Figure 11.5 The tailored database.

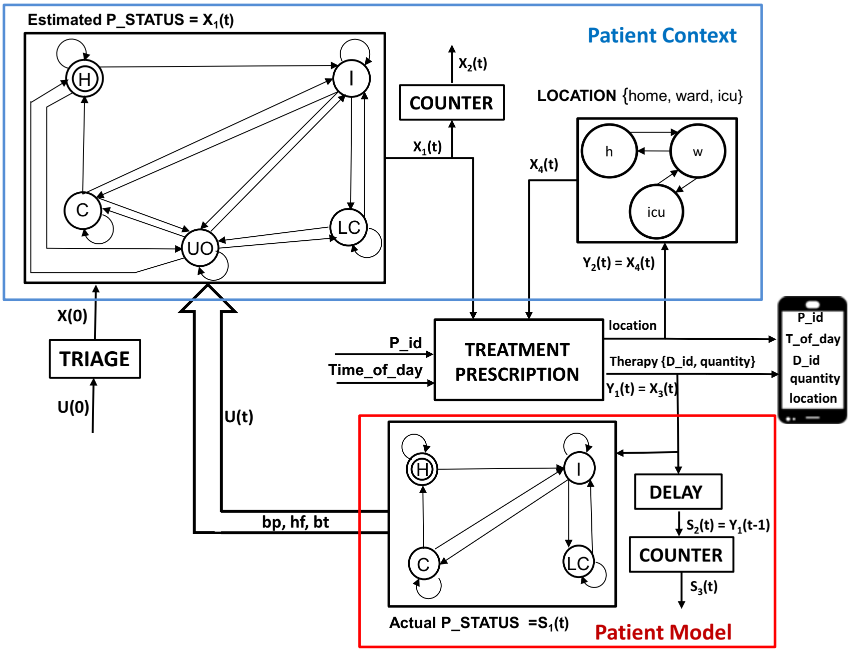


Figure 11.6 The system structure.

Listing 11.2 Querying the nurse's device.

```
select D_id, quantity, location
    from patient, therapy, made_of, D_id
    where P_id = "pp" AND time_of_day = "hh:mm"
```

In Fig. 11.6 the global system structure is represented showing the Moore state diagrams of the estimated and the actual patient status, and of the location respectively, as it will be detailed in Section 11.4.

11.4 The Boolean Control Network system model

We are now in a position to introduce the BCN models for our case study.

11.4.1 The patient context model

Let us first consider the patient context model. We assume as input vector the *3*-dimensional vector $U(t)$, where:

$U_1(t)$ denotes the (*low*, *medium*, or *high*) value of the body temperature (*bt*) at time *t*;

$U_2(t)$ denotes the (*low*, *medium*, or *high*) value of the body pressure (*bp*) at time t;

$U_3(t)$ denotes the (*low*, *medium*, or *high*) value of the heart frequency (*hf*) at time t.

The corresponding canonical vector, $u(t)$, therefore belongs to $\mathscr{L}_{27}$, since each variable $U_i(\cdot)$, $i = 1, 2, 3$, can take three distinct values.

The state variable $X(t)$ is a *4*-dimensional vector, where:

- $X_1(t)$ denotes the Estimated Patient status (in other words, the diagnosis) at time t with respect to a specific form of illness: it takes values in the set $\{H, C, UO, I, LC\}$.
- $X_2(t)$ represents a counter variable, that keeps track of how many consecutive times up to time t the estimated patient status has remained invariant. In other words, $X_2(t) = m$ if $X_1(t) = X_1(t-1) = \ldots = X_1(t-m+1)$, but $X_1(t-m+1) \neq X_1(t-m)$. To ensure that X_2 takes values in a finite set, and for the sake of simplicity,[1] we assume that we keep track until $X_2(t)$ reaches the value *3*, and then we stop. This amounts to saying that $X_2(t)$ belongs to $\{1, 2, \geq 3\}$.
- $X_3(t)$ is the prescribed therapy at time t, belonging to a finite set, say $\{Th0, Th1, \ldots, Th5\}$, where $Th0$ means that the patient does not receive any drug.
- $X_4(t)$ is the prescribed location (*home, ward, ICU*) where the patient will get the therapy at time t.

The corresponding canonical representation, $x(t)$, under the previous assumptions will belong to $\mathscr{L}_{270}$, since $270 = 5 \times 3 \times 6 \times 3$. Finally, we assume as output of the Patient context the 2-dimensional vector $Y(t)$, where:

$Y_1(t)$ is the prescribed therapy at time t;

$Y_2(t)$ is the prescribed location (*home, ward, ICU*) where the patient will get the therapy at time t.

Clearly, $Y_1(t) = X_3(t)$ and $Y_2(t) = X_4(t)$. Moreover, the canonical representation of $Y(t)$, $y(t)$, belongs to $\mathscr{L}_{18}$, since 18 is the number of possible combinations of therapies and locations. Note, however, that the set of possible outputs can be significantly reduced: for instance, the location *home* is compatible only with the choice to dismiss the patient, after considering his/her health status, and with prescribed therapies such as $Th0$ (no drugs) or a light therapy (say, $Th1$). At the same time certain therapies can be administered only when the patient is in the *ICU*. So,

[1] All the numbers used in this context are, of course, arbitrary and meant to purely exemplify how to design the algorithm and to convert it into a BCN.

one may reasonably assume that a good number of the 18 output values are not realistic and hence can be removed, thus reducing the size of $y(\cdot)$.

It is worthwhile to introduce a few comments about the initial state $X(0)$ (or its canonical representation $x(0)$) and about the update of the state variables $X_i(t)$, $i = 1, \quad 2, \quad 3, \quad 4$. The initial state can be regarded as the result of the triage process: when patients are admitted to the *Emergency Room*, a preliminary diagnosis is made, based on the three measures $U_1(0), U_2(0)$, and $U_3(0)$, since there may be no previous history of the patient and the hospital admission requires a fast evaluation of the medical conditions of the patient. So, $X_1(0)$ may be a static function of $U(0)$. $X_3(0)$ is automatically set to $Th0$, while $X_2(0)$ is set to 1 and $X_4(0)$ to *home*.

We note that $X_1(t+1)$ is naturally expressed as a logic function of $X_1(t), X_2(t), X_3(t), X_4(t)$ and $U(t)$, say $X_1(t+1) = f_1(X(t), U(t))$. On the other hand, $X_2(t+1)$ naturally depends on $X_2(t), X_1(t)$ and $X_1(t+1)$, and, since we have just pointed out that $X_1(t+1) = f_1(X(t), U(t))$, we can in turn express $X_2(t+1)$ as $X_2(t+1) = f_2(X(t), U(t))$. Similarly, $X_3(t+1)$ and $X_4(t+1)$ are functions of $X_1(t+1)$, $X_2(t+1)$, $X_3(t)$, $X_4(t)$, and $U(t)$, and hence can be expressed, in turn, as functions of $X_1(t), X_2(t), X_3(t), X_4(t)$, and $U(t)$. On the other hand, as we previously remarked, $Y_1(t) = X_3(t)$ and $Y_2(t) = X_4(t)$. This implies that

$$X(t+1) = f(X(t), U(t)),$$

while

$$Y\left(t\right) = \begin{bmatrix} X_3(t) \\ X_4(t) \end{bmatrix},$$

and hence

$$x(t+1) = L \ltimes u(t) \ltimes x(t),$$

$$y(t) = Mx(t), \cdots \to t \in \mathbb{Z}_+$$

for suitable choices of the logical matrices $L \in \mathscr{L}_{270 \times (27 \cdot 270)}$ and $M \in \mathscr{L}_{18 \times (27 \cdot 270)}$.

11.4.2 The patient model

At this point we consider the patient model. A reasonable choice of the patient state variables is the following one:

- $S_1(t)$ represents the actual patient status that takes values in the set $\{H, C, I, LC\}$. Note that this is a proper subset of the set where the

Estimated Patient status takes values, since of course the value UO in this case does not make sense.

- $S_2(t)$ represents the therapy that has been prescribed at time $t-1$, and hence it coincides with $Y_1(t-1)$.
- $S_3(t)$ is a counter variable that keeps track of how many consecutive times up to time t the therapy has remained invariant. In other words, $S_3(t)=m$ if $S_2(t)=S_2(t-1)=\ldots=S_2(t-m+1)$, but $S_2(t-m+1)\neq S_2(t-m)$. Also in this case, we put a bound on m and assume that $S_3(t)$ belongs to $\{1,2,\geq 3\}$.
- Finally, $S_4(t)$ is the vector collecting the measures of the vital parameters at time $t-1$, namely $S_4(t)=U(t-1)$.

For the patient model the natural input is $Y(t)$ (in fact, $Y_1(t)$ could suffice), while the output is $U(t)$. Since $U(t)$ is the patient vital parameters at time t, it is reasonable to assume that these measures depend on their values at time $t-1$ (and hence on $S_4(t)$), on the patient status $S_1(t)$, the given therapy at time $t-1$, $S_2(t)$ (indeed it is not realistic to assume that the effect of the therapy is instantaneous), and on the duration of the therapy, namely, on $S_3(t)$.

Following a similar reasoning to the one adopted for the patient context model, we can claim that the patient model is described by the logic equations

$$S(t+1)=f_p(S(t),Y(t),U(t)),$$

$$U(t)=h_p(S(t)),$$

and hence by the BCN

$$s(t+1)=F\ltimes y(t)\ltimes u(t)\ltimes s(t),$$

$$u(t)=Hs(t),\ t\in\mathbb{Z}_+,$$

for suitable choices of the logical matrices $F\in\mathscr{L}_{1944\times(18\cdot 27\cdot 1944)}$ and $H\in\mathscr{L}_{27\times 1944}$, since $1944=4\cdot 6\cdot 3\cdot 27$. So, to summarize, we have the following two models:

$$x(t+1)=L\ltimes u(t)\ltimes x(t), \tag{11.4}$$

$$y(t)=Mx(t),\ t\in\mathbb{Z}_+ \tag{11.5}$$

and

$$s(t+1)=F\ltimes y(t)\ltimes u(t)\ltimes s(t), \tag{11.6}$$

$$u(t) = Hs(t), t \in \mathbb{Z}_+ \tag{11.7}$$

For the sake of simplicity, we will use the following notation: $270 = \dim x =: N_x$, $1944 = \dim s =: N_s$, $27 = \dim u =: N_u$ and $18 = \dim y =: N_y$. If we replace (11.7) and (11.5) in (11.6) and keep into account that

$$s(t) \ltimes s(t) = \Phi s(t),$$

where $\Phi \in \mathscr{L}_{N_s^2 \times N_s}$ is a logical matrix known as *power-reducing matrix* (Cheng et al., 2011), then (11.6) becomes

$$s(t+1) = F \ltimes M \ltimes x(t) \ltimes H \ltimes \Phi \ltimes s(t). \tag{11.8}$$

At the same time, we can swap, namely, reverse the order of, the vector $x(t)$ and the vector $H \ltimes \Phi \ltimes s(t)$ by resorting to the swap matrix W of suitable size (Cheng et al., 2011), thus obtaining

$$s(t+1) = F \ltimes M \ltimes W \ltimes H \ltimes \Phi \ltimes s(t) \ltimes x(t) = A(s(t) \ltimes x(t)), \tag{11.9}$$

where

$$A := F \ltimes M \ltimes W \ltimes H \ltimes \Phi \in \mathscr{L}_{N_s \times N_s N_x}$$

Similarly, if we replace (11.7) in (11.4), we get:

$$x(t+1) = L \ltimes H \ltimes s(t) \ltimes x(t) = B(s(t) \ltimes x(t)), \tag{11.10}$$

where

$$B := L \ltimes H \in \mathscr{L}_{N_x \times N_s N_x}.$$

Now, the overall model, keeping into account both the patient context and the patient model, becomes:

$$s(t+1) = A \ltimes s(t) \ltimes x(t), \tag{11.11}$$

$$x(t+1) = B \ltimes s(t) \ltimes x(t). \tag{11.12}$$

If we introduce the status of the overall system

$$v(t) := s(t) \ltimes x(t) \in \mathscr{L}_{N_s N_x},$$

we get

$$v(t+1) = (A \ltimes v(t)) \ltimes (B \ltimes v(t)).$$

It is a matter of elementary calculations to verify that once we denote by a_i the ith column of A and by b_j the jth column of B, the previous equation can be equivalently rewritten as

$$v(t+1) = Wv(t), \tag{11.13}$$

where

$$W := \begin{bmatrix} a_1 \ltimes b_1 & a_2 & \ltimes & b_2 & \dots & a_{N_sN_x} \ltimes b_{N_sN_x} \end{bmatrix} \in \mathscr{L}_{N_sN_x \times N_sN_x}.$$

In addition, one can assume as system output

$$y(t) = Mx(t)$$

that can be rewritten as

$$y(t) = \Psi v(t), \tag{11.14}$$

where

$$\Psi := \begin{bmatrix} M & M & \dots & M \end{bmatrix} \in \mathscr{L}_{N_y \times N_sN_x}.$$

So, Eqs. (11.13) and (11.14) together describe a BN that models the overall closed-loop system.

11.5 Real-life properties and their mathematical formalization

In this section, we investigate the properties of the overall system, obtained by the feedback connection of the patient context and of the patient model, namely, the BN (11.13) and (11.14).

As stated in Section 11.2, we aim at providing general ideas about the mathematical properties of the system that have a clear practical relevance in this context, rather than checking those properties for a specific choice of the logical matrices involved in the system description. Thus we shall not provide numerical values for the quadruple of logical matrices (L, M, F, H), but *we shall show how to reduce our specific feedback system (or parts of it) to standard set-ups for which these properties have already been investigated*.

Thus the purpose of this section is to illustrate how issues regarding the correct functioning of the system, the possibility of identifying its "real state" from the available measurements, etc., that mathematically formalize the natural requirements on a closed-loop context-aware system describing a medical application (but not only!), can be easily addressed in the context of BCNs.

11.5.1 Identifiability of the patient status

A first question that is meaningful to pose is whether the Patient model is a good one, namely it will lead to the correct functioning of the overall system. To clarify what we mean when posing this question, we first need to better explain the perspective we have taken in modeling the patient. We have assumed that the patient is in a certain medical condition with respect to a specific medical problem. Thus the diagnosis pertains only to the level/seriousness of the patient's health condition, and not to the specific cause of the illness. Such a medical condition is revealed by the fact that patient vital parameters (*bp, bt, hf*), namely, the patient output $U(t)$, take values outside of the "normal range." The medical status is of course affected by the therapy Y and can be associated with different values of U, so the output measure $U(t)$ at time t together with the therapy $Y(t)$ (or $Y_1(t)$) do not allow to uniquely determine $S(t)$. In addition, some therapies may need some time to become effective (which is the reason why we introduced the state variable $S_2(t)$). On the other hand, a good (deterministic) model of the patient[2] necessarily imposes that the measured vital parameters are significant and hence allow physicians to determine the actual patient status after a finite number of observations. From a mathematical point of view, this amounts to assuming that the patient model (11.6) and (11.7) is reconstructable, namely, there exists $T \in \mathbb{Z}_+$ such that the knowledge of the signals $u(\cdot)$ and $y(\cdot)$ in $[0, T]$ allows to uniquely determine $s(T)$. Specifically, we have the following:

Definition 1 The BCN (11.6) and (11.7), with $s(t) \in \mathcal{L}_{N_s}, u(t) \in \mathcal{L}_{N_u}$ and $y(t) \in \mathcal{L}_{N_y}$, is said to be *reconstructable* if there exists $T \in \mathbb{Z}_+$ such that the knowledge of the input and output vectors in the discrete interval $\{0, 1, ..., T\}$ allows to uniquely determine the final state $s(T)$.

It is worth noticing that the BCN (11.6) and (11.7) is different from the standard ones for which the observability and reconstructability problems have been addressed in the literature (see Fornasini & Valcher, 2013a; Laschov et al., 2013; Zhang & Zhang, 2016), since this BCN is intrinsically in a closed-loop condition, as the BCN output u(t) affects the state update at time $t+1$. However, by replacing (11.7) in (11.6), and by using again the power-reducing matrix, we can obtain:

$$s(t+1) = F \ltimes y(t) \ltimes H \ltimes \Phi \ltimes s(t),$$
$$u(t) = Hs(t), t \in \mathbb{Z}_+,$$

[2] As previously mentioned, we have adopted a deterministic model and assumed that everything works according to statistics and well-settled procedures: therapies are designed according to specific protocols and statistically lead to the full recovery of the patient. This is the reason why the possibility that the patient dies is not contemplated.

which, in turn, can be rewritten as

$$s(t+1) = \mathscr{F} \ltimes y(t) \ltimes s(t), \tag{11.15}$$

$$u(t) = Hs(t), t \in \mathbb{Z}_+, \tag{11.16}$$

where

$$\mathscr{F} \; := \left[\mathscr{F}_1 \quad \mathscr{F}_2 \quad . \quad . \quad . \quad \mathscr{F}_{N_s} \right]$$

and

$$\mathscr{F}_i := \left[f_i \ltimes (H \ltimes \Phi)\delta_{N_s}^1 \quad \ldots \quad f_i \ltimes (H \ltimes \Phi)\delta_{N_s}^{N_s} \right],$$

where we have denoted by f_i the ith column of the matrix F. This allows to reduce the reconstructability problem for this specific BCN to a standard one, for which there are lots of results and algorithms (see Fornasini & Valcher, 2013a; Zhang & Johansson, 2020; Zhang & Zhang, 2016; Zhang et al., 2019).

Clearly, the matrices F and H must be properly selected to guarantee the reconstructability of the Patients' status. This means, in particular, that *the vital parameters to measure must be chosen in such a way that they are significant enough to allow to identify the actual medical conditions of the patient.*

From a less formal viewpoint, it is worth underlying that the reconstructability problem reduces to the problem of correctly identifying the state variable $s_1(t)$, since the definition of $s_i(t), \quad i = 1, 2, 3, 4$, allows to immediately deduce that such values can be uniquely determined from the variables $y_1(t)$ and $u(t)$. So, one could focus on a lower dimension model expressing $s_1(t+1)$ in terms of $s_i(t)$, $i = 1, 2, 3, 4, u(t)$ and $y_1(t)$, where $s_i(t), \quad i = 1, 2, 3, 4, \; u(t)$ and $y_1(t)$ are known and address the reconstructability of $s_1(t)$ from $u(t)$, assuming $s_i(t), \quad i = 1, 2, 3, 4$, and $y_1(t)$ as inputs.

11.5.2 Correct diagnosis

Of course, once we have ensured that the patient model (11.6) and (11.7) is reconstructable, and hence we have properly chosen the vital parameters to measure to identify the patient status, the natural question arises: *Is the patient context correctly designed so that after a finite (and possibly small) number of steps T, the patient status $s_1(t)$ and the estimated patient status $x_1(t)$ coincide for every $t \geq T$?* This amounts to saying that the protocols to evaluate the Patient Status have been correctly designed.

To formalize this problem, we need to introduce a comparison variable, say $z(t)$. This variable takes the value δ_2^1 (namely the unitary or YES value) if $s_1(t) = x_1(t)$ and the value δ_2^2 (namely, the zero or NO value) otherwise. Keeping in mind that $S_1(t)$ takes values in $\{H, C, \quad I, \quad LC\}$ (and hence $s_1(t) \in \mathscr{L}_4$), while $X_1(t)$ takes values in $\{H, C, UO, I, LC\}$ (and hence $x_1(t) \in \mathscr{L}_5$), this leads to

$$z(t) = \begin{bmatrix} C_1 & C_2 & C_3 & C_4 \end{bmatrix} \ltimes s_1(t) \ltimes x_1(t),$$

where[3] $C_i \in \mathscr{L}_{2\times 5}$ for every $i \in [1, 4]$. Moreover,

C_1 is the block whose first column is δ_2^1 while all the others are δ_2^2;
C_2 is the block whose second column is δ_2^1 while all the others are δ_2^2;
C_3 is the block whose fourth column is δ_2^1 while all the others are δ_2^2;
C_4 is the block whose fifth column is δ_2^1 while all the others are δ_2^2.

Clearly, $z(t)$ can also be expressed as a function of $s(t)$ and $x(t)$ and hence as a function of $v(t)$. This leads to

$$z(t) = \mathbb{C}v(t),$$

for a suitable $\mathbb{C} \in \mathscr{L}_{2\times N_s N_x}$ Thus *the problem of understanding whether the system is designed to produce the correct diagnosis can be equivalently translated into the mathematical problem of determining whether for every initial condition*, $v(0)$, *the output trajectory of the system*

$$v(t+1) = Wv(t) \tag{11.17}$$

$$z(t) = \mathbb{C}v(t) \tag{11.18}$$

eventually takes the value δ_2^1. In other words, we need to ensure that there exists $t \in \mathbb{Z}_+$ such that, for every $v(0) \in \mathscr{L}_{N_s N_x}$, the corresponding output trajectory $z(t), t \in \mathbb{Z}_+$, satisfies $z(t) = \delta_2^1$, for every $t \geq T$. Note that the idea is that once the seriousness level of the patient illness has been correctly diagnosed, this information will never be lost, even if the patient health status will change.

An alternative approach to this problem is to define the set of states

$$CD := \left\{ v(t) \in \mathscr{L}_{N_s N_x} : s_1(t) \ltimes x_1(t) \in \left\{ \delta_4^1 \ltimes \delta_5^1, \delta_4^2 \ltimes \delta_5^2, \delta_4^3 \ltimes \delta_5^4, \delta_4^4 \ltimes \delta_5^5 \right\} \right\},$$

[3] To improve the notation one could sort the set of values of the estimated patient's status as follows: $\{H, C, \quad I, \quad LC, UO\}$. In this way, each of the blocks C_i would have the ith column equal to δ_2^1 and all the remaining ones equal to δ_2^2.

that represent all possible situations where the estimated patient status $x_1(t)$ coincides with the patient status $s_1(t)$ (in other words, CD is the set of correct diagnoses) and to impose that such a set is *a global attractor* of the system. From a formal point of view, the set CD is a *global attractor of the BN* (11.17) if there exists $T \geq 0$ such that for every $v(0) \in \mathcal{L}_{N_s N_x}$, the corresponding state evolution $v(t)$, $t \in \mathbb{Z}_+$, of the BN (11.17) belongs to CD for every $t \geq T$.

This property can be easily checked (Cheng et al., 2011; Fornasini, & Valcher, 2013b) by simply evaluating that all rows of $W^{N_s N_x} \in \mathcal{L}_{N_s N_x} \times \mathcal{L}_{N_s N_x}$, the $N_s N_x$ power of W, are zero except for those whose indexes correspond to the canonical vectors in CD.

11.5.3 Successful therapies

As previously mentioned, when modeling the evolutions of the patient context and the patient model in a deterministic way, we are describing the evolution of the average case of a patient affected by a specific form of illness. Accordingly, as mentioned in Section 11.2.1, we are giving certain interpretations to the patient symptoms, as captured by the values of his/her vital parameters, and based on them we are applying well-settled medical protocols to prescribe therapies and locations where such therapies need to be administered. In this context it is clear that death is not contemplated, since this would correspond to assuming that a given medical protocol deterministically leads to the death of the patient and this does not make sense. Similarly, a protocol that deterministically leads to an equilibrium state where the Patient status is C, I or LC is not acceptable. In other words, *the only reasonable solution is to have designed the Patient Context in such a way that (1) the patient status is eventually H; (2) the estimated patient status is, in turn, H.*

Conditions (1) and (2) correspond to constraining *the global attractor of the system evolution to be a proper subset, say* $\mathcal{H}$*, of the set* CD we previously defined. Specifically, we define the set $\mathcal{H}$ as follows:

$$\mathcal{H} = \{v(t) \in \mathcal{L}_{N_s N_x} : s_1(t) \ltimes x_1(t) = \delta_4^1 \ltimes \delta_5^1\},$$

that represent all possible situations where the estimated patient status $x_1(t)$ is healthy and it coincides with the patient status $s_1(t)$ (in other words, $\mathcal{H}$ is the set of states corresponding to a healthy patient whose health status has been correctly identified), and *to impose that such a set is a global attractor of the system.*[4]

Also, in this case, it is possible to verify whether such a requirement is met by evaluating if all rows of $W^{N_s N_x} \in \mathcal{L}_{N_s N_x} \times \mathcal{L}_{N_s N_x}$, the $N_s N_x$ power of W, are zero except for those whose indexes correspond to the canonical vectors in $\mathcal{H}$.

11.6 Evaluation and conclusions

In this paper we introduced a novel methodology, which uses well-established systems theory tools, to formally assess some safety properties of feedback context-aware database systems. As a proof of concept, we have used an interesting case study, related to the evolution of the health status of a patient, to illustrate how a feedback context-aware system can be modeled by means of a BCN. Indeed, the patient is subjected to medical therapies and his/her vital parameters are not only the outcome of the therapies, but also the input based on which therapies are prescribed. By making use of a simplified and deterministic logical model, expressed in terms of BCNs/BNs, we have been able to illustrate how the most natural practical goals that the overall closed-loop system needs to achieve may be formalized, and hence investigated, by resorting to well-known systems theory concepts. Clearly, the given model can be improved and tailored to the specific needs, to account for more complicated algorithms and more exhaustive sets of data, but the core ideas have already been captured by the current model. Also, we have addressed what seemed to be the most natural targets in the specific context, but different or additional properties may be investigated, in case the same modeling technique is applied to describe closed-loop context-aware systems of different nature.

The use of a deterministic model of the patient health evolution, to plan therapies based on measured vital parameters, represents a first step

[4] Note that we are not introducing additional constraints, in particular we are assuming that the vital parameters u of the patient can change within the set of values compatible with a healthy status. Of course, one could further constrain the set $\mathcal{H}$ by assuming that the prescribed therapy is $Th0$, the patient is at home, and all the counters have reached the saturation level. Even in this case, we may regard as acceptable the existence of a limit cycle, since this would only correspond to oscillations of the values of the state variable s_4 within a small set of values that do not raise any concern. Clearly, one may impose also for s_4 and hence for u a prescribed desired value, and this would mean asking that the system has a single *equilibrium point* (the set $\mathcal{H}$ has cardinality one) which is a global attractor.

toward the design of an accurate algorithm to employ in the mobile device of a nurse.

A final question deals with the computational cost of the procedure and here the bad news comes. Many studies have established that verifying properties such as observability, controllability, and stabilizability of BCNs are NP-hard in the number of nodes (Weiss et al., 2018; Zhang & Johansson, 2020); however, in some cases, the computational complexity can be reduced (Lu et al., 2019; Zhao et al., 2016) and it will not exceed $O(N^2)$ with $N = 2^n$, where n is the number of state variables in a BCN (Zhu et al., 2019). Thus further research is needed to find meaningful modularizations of the BCN into sets of BCNs with a smaller number of state variables each, which can then be reassembled into the global system.

Furthermore, a probabilistic model, together with some warning system that advises the nurse of when different decisions are possible with different confidence levels, and hence there is the need for the immediate supervision of a specialist, is the target of future research.

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